Understanding Vector Calculus

Jerrold Franklin

Preface

My purpose in this development of vector calculus is to present it in a way that will prove useful to you for the rest of your career in science or mathematics. You can think of this presentation as a 'workbook' for learning how to use vector calculus in practical calculations and derivations. After studying the text and doing the problems, you should not have to memorize long equations or need to look anything up while you work in your field. I want you to get to the point where you can treat the use of vector calculus on the same level as you would simple algebraic calculations, working them out as you go.

The necessary background for using this book is just a knowledge of one-dimensional differential and integral calculus. Having had a multivariable calculus course is unnecessary because the multivariable aspect of vector calculus will be developed from scratch. In fact, you might have to unlearn some of the things covered in a multivariable calculus course if it relied too much on using coordinate systems rather than the vector methods we will introduce.

Although the examples in the book are usually related to physical examples, particularly electromagnetism, it is not necessary to have any background in physics. If any of the physics is new to you, you can simply disregard it and concentrate on the mathematics. On the other hand, if you have a background in (relatively simple) physics, I think you will find the references to physics of interest. In treating electromagnetism, we use what are sometimes called 'natural units' in which the potential of a point charge q is given by q/r. In this way, we avoid the introduction of artificial constants that might complicate the mathematics.

The book has two parts. Part one is a brief text developing vector calculus from the very beginning, and then including some more detailed applications.

Part two consists of answered problems, which are all closely related to the development of vector calculus in the text. Although there are answers immediately following each problem, you should not be too quick to use the answer as a crutch. For problems that involve working through a calculation, please try your hardest to do it on your own. Then you can use the answer to check your result, or possibly see another way of doing the problem. If you go immediately to the answer, you will be learning about vector calculus but not how to use vector calculus. Of course, if you don't see how to start, or run into a roadblock, the answer is there to help you.

There are some problems asking you to derive vector calculus results using Cartesian coordinates. I have to admit that you probably shouldn't go through all that algebra. Those problems are just there to show you how complicated things can get. There are also some problems involving derivations. You should probably think about how you would proceed, and then look at the answer because the vector derivation will usually be much different and simpler than you might have seen in a multivariable calculus course.

In any event, I hope you find my treatment interesting, perhaps entertaining, and above all useful. I think you will see that you have nothing to fear from vector calculus, and that it will prove to be a good friend for the rest of your life. Feel free to use me as a resource, either by answering questions you might have about the material in the book, or about anything else. You can contact me at Jerry.F@Temple.edu.